# Sensorless Direct Torque Control of PMSM Drive with EKF Estimation of Speed, Rotor Position and Load Torque Observer

Said Benaggoune, Sebti Belkacem and Rachid Abdessemed Leb Research Laboratory, Department of Electrical Engineering, University of Banta, Street Med El Hadi Boukhlouf, Batna 05000, Algeria

**Abstract:** This study presents the estimation of speed, rotor position and load torque of a robust sensorless Direct Torque Control (DTC) method for Permanent Magnet Synchronous Motor (PMSM) based on the Extended Kalman Filter (EKF). The model of PMSM and its EKF models in MATLAB/Sirnulink simulation environment are developed. The proposed EKF speed position estimation method is also proved insensitive to the PMSM parameter variations. Simulation results demonstrate a good performance and robustness.

**Key words:** DTC, Extended Kalman Filter (EKF), Permanent Magnet Synchronous Motor (PMSM), control, sensorless, load torque observer

## INTRODUCTION

The Permanent Magnet Synchronous Motor (PMSM) is an ideal candidate for high-performance industrial drives since it features simple structure, high-energy efficiency, reliable operation and high power density. In the conventional PMSM drive systems, speed and torque control is achieved by obtaining the rotor position or speed information through shaft sensors such as optical encoders, Hall-Effect sensors. The use of such sensors increases the complexity, weight and cost of the system and reduces the overall reliability of the controlled drive system. Speed position sensorless control of motor drive systems overcomes the above shortcomings and improves the overall system reliability, ruggedness dynamic performance (Zhu et al., 2000). The Extended Kalman Filter is an optimal stochastic observer in the least-square sense for estimating the states of dynamic non-linear systems and provides optimal filtering of the noises in measurement and inside the system if the covariances of these noises are known. Hence it is a viable candidate for the on-line determination of the rotor position and speed of a PMSM. However, the practical industrial application of EKF sensorless motor control has not been reported. The technical problems include: A detailed dynamic model for the PMSM including initial rotor position is not available; formulation of the EKF model in closed form is not available; discrete time model of the overall controlled system and details of power electronics Since its introduction in 1985, the Direct Torque Control (DTC), (Canudas, 2000; Leite et al., 2004) principle was widely used for PMSM drives with fast dynamics. Despite its simplicity, DTC is able to produce very fast torque and flux control, if the torque and the flux are correctly estimated. It is robust with respect to motor parameters and perturbations. Recently, the emphasis of research on PMSM has been on sensorless drive, which eliminates flux and speed sensors mounted on the motor. In addition, the development of effective speed and flux estimators has allowed good Rotor Flux-Oriented (RFO) performance at all speeds except those close to zero. Sensorless control has improved the motor performance, compared to the Volts/Hertz (or constant flux) controls, (Kosaka and Uda, 2004). The EKF is considered to be suitable for use in high-performance induction motor drives and it can provide accurate speed estimates in a wide speed-range, including very low speed (Akin, 2003; Shen et al., 2002).

The contribution of this study consists of the development of an EKF based speed sensorless DTC system for an improved performance, especially against variations in the load torque. The developed EKF algorithm involves the estimation of speed, rotor position and load torque. The performance of the control system with the proposed EKF algorithm has been demonstrated with simulations using MATLAB/Sirnulink.

# MODELING OF THE PMSM

The electrical and mechanical equations of the PMSM in the rotor reference (dq) frame as expressed as follows, (Canudas, 2000):

$$\begin{cases} \frac{dI_{d}}{dt} = -\frac{R_{s}}{L_{d}} + \frac{L_{q}}{L_{d}} p \omega_{r} I_{q} + \frac{1}{L_{d}} U_{d} \\ \frac{dI_{q}}{dt} = -\frac{R_{s}}{L_{q}} - \frac{L_{d}}{L_{q}} p \omega_{r} I_{d} - \frac{\Phi_{f}}{L_{q}} p \omega_{r} + \frac{1}{L_{q}} U_{q} \\ \frac{d\omega_{r}}{dt} = \frac{3p}{2J} [(L_{d} - L_{q})I_{q}I_{d} + \Phi_{f}I_{q}] - \frac{1}{J} T_{L} - \frac{f}{J} \omega_{r} \end{cases}$$
(1)

The space-state equations of the PMSM can be written as:

$$\frac{\mathrm{d}}{\mathrm{dt}} [X] = [A] [X] + [B] [U]$$
 (2)

Defining:

$$\begin{bmatrix} \mathbf{X} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{\mathbf{d}} & \mathbf{I}_{\mathbf{q}} \end{bmatrix}^{\mathbf{T}}$$

$$\begin{bmatrix} \mathbf{V} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{\mathbf{d}} & \mathbf{U}_{\mathbf{q}} & \Phi_{\mathbf{f}} \end{bmatrix}^{\mathbf{T}}$$
(3)

$$\begin{bmatrix} \mathbf{A} \end{bmatrix} = \begin{bmatrix} -\frac{\mathbf{R_s}}{\mathbf{L_d}} & \boldsymbol{\omega_r} & \frac{\mathbf{L_q}}{\mathbf{L_d}} \\ -\boldsymbol{\omega_r} & \frac{\mathbf{L_d}}{\mathbf{L_q}} & -\frac{\mathbf{R_s}}{\mathbf{L_q}} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{B} \end{bmatrix} = \begin{bmatrix} \frac{1}{\mathbf{L_d}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\mathbf{L_q}} & -\frac{\boldsymbol{\omega_r}}{\mathbf{L_q}} \end{bmatrix}$$

$$(4)$$

The equation of the electromagnetic torque is given by:

$$\begin{cases} T_{L} = \frac{3p}{2} [(L_{d} - L_{q})I_{q}I_{d} + \Phi_{f}I_{q}] \\ J\frac{d\Omega}{dt} = T_{L} - T_{r} - f \Omega \end{cases}$$
 (5)

## BASIC DTC PRINCIPLES

DTC is a control philosophy exploiting the torque and flux producing capabilities of ac machines when fed by a voltage source inverter that does not require current regulator loops, still attaining similar performances to that obtained by a vector control drive.

**Behavior of stator flux:** In the reference  $(\alpha, \beta)$ , the stator flux can be obtained by the following equation:

$$\overline{V}_{s} = R_{s}\overline{I}_{s} + \frac{d}{dt}\overline{\Psi}_{s}$$
 (6)

Table 1: Selection table for direct torque control							
$\Delta\Psi_s$	$\Delta\Gamma_{ m e}$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
1	1	$V_2$	$V_3$	$V_4$	$V_5$	$V_6$	$V_1$
	0	$V_7$	$V_0$	$V_7$	$V_0$	$V_7$	$V_0$
	-1	$V_6$	$V_1$	$V_2$	$V_3$	$V_4$	$V_5$
0	1	$V_3$	$V_4$	$V_5$	$V_6$	$V_1$	$V_2$
	0	$V_0$	$V_7$	$V_0$	$V_7$	$V_0$	$V_7$
	-1	$V_5$	$V_6$	$V_1$	$V_2$	$V_3$	$V_4$

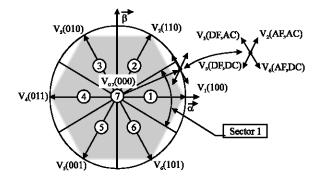


Fig. 1: Partition of the complex plan in six angular

By neglecting the voltage drop due to the resistance of the stator to simplify the study (for high speeds), we find:

$$\overline{\Psi}_{s} \approx \overline{\Psi}_{s0} + \int_{0}^{t} \overline{V}_{s} dt$$
 (7)

For one period of sampling, the voltage vector applied to the PMSM remains constant, we can write:

$$\overline{\Psi}_{s}(k+1) \approx \overline{\Psi}_{s}(k) + \overline{\nabla}_{s}T_{e}$$
 (8)

**Behavior of the torque:** The electromagnetic torque is proportional to the vector product between the stator and rotor flux according to the following expression (Canudas, 2000):

$$T_{L} = k(\overline{\Psi}_{s} \times \overline{\Psi}_{s}) = k |\overline{\Psi}_{s}| |\overline{\Psi}_{r}| \sin(\delta)$$
 (9)

**Development of the commutation strategy:** Table 1, shows the commutation strategy suggested by Takahashi and Noguchi (1986) to control the stator flux and the electromagnetic torque of the PMSM.

Figure 1 gives the partition of the complex plan in six angular sectors  $S_{1=1...6}$ .

IT : Increase the Torque,DT : Decrease the Torque.IF : Increase the Flux,DF : Decrease the Flux.

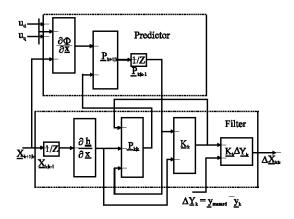


Fig. 2: Structure of Kalman filter estimator

## DEVELOPMENT OF THE EKF ALGORITHM

The Kalman filter is a well-known recursive algorithm that takes the stochastic state space model of the system together with measured outputs to achieve the optimal estimation of states (Terric and Jadric, 2001). The optimality of the state estimation is achieved with the minimization of the mean estimation error. In this study, EKF is used for the estimation of  $I_{\rm ds},\,I_{\rm qs},\,\omega_{\rm f},\,\theta$  and  $T_{\rm L}$ .

Figure 2 shows the structure of a Kalman filter.

The discrete model of the PMSM can be given as follows:

$$\begin{cases} x(k+1) = f(x(k), u(k)) + w(k) \\ y(k) = c(x(k)) + v(k) \end{cases}$$
(10)

Where w(k) the measurement noise and v(k) is the process noise.

# APPLICATION OF THE EXTENDED KALMAN FILTER

The speed estimation algorithm of the extended Kalman filter can be simulated by the MATLAB/Simulink software, which consists of an S-Function block as shown in Fig. 3 (Shen *et al.*, 2002).

**Prediction of the state vector:** Prediction of the state vector at sampling time (k+1), from the input u(k), state vector at previous sampling time x(k/k).

$$\hat{\mathbf{x}}(\mathbf{k} + 1/\mathbf{k}) \triangleq \mathbf{F}(\hat{\mathbf{x}}(\mathbf{k}/\mathbf{k}), \mathbf{u}(\mathbf{k})) \tag{11}$$

**Prediction covariance computation:** The prediction covariance is updated by:

$$P(k + 1/k) = F(k)P(k)F(k)^{T} + Q$$
 (12)

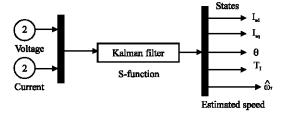


Fig. 3: Simulink model of EKF speed estimation.

Where Q is the covariance matrix of the system noise,

$$F(k) = \frac{\partial f}{\partial x}\Big|_{x(k) = \tilde{x}(k/k)}$$
(13)

**Kalman gain computation:** The Kalman filter gain (correction matrix) is computed as:

$$L(k+1) = P(k+1/k).C(k)^{T}.$$

$$(C(k)P(k+1/k)C(k)^{T} + R)^{-1}$$
(14)

With:

$$C(k) = \frac{\partial c(x(k))}{\partial x(k)} \Big|_{x(k) = \hat{x}(k)}$$

**State vector estimation:** The predicted state-vector is added to the innovation term multiplied by Kalman gain to compute state-estimation vector. The state-vector estimation (filtering) at time (k) is determined as:

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + L(k+1)(y(k+1) - C\hat{x}(k+1/k))$$
(15)

## PROPOSED SENSORLESS PMSM DRIVE

The proposed sensorless PMSM drive is shown in Fig. 4. The stator flux is estimated by the EKF and used in the DTC control. The machine parameters are listed in Table 2.

# SENSITIVITY STUDY AND SIMULATION RESULTS

Computer simulations have been carried out in order to validate the effectiveness of the proposed scheme. The speed, current, rotor position and load torque responses are observed under various operating conditions such as change in reference speed, step change in load and parameter variation.

During the simulations, the torque set value is limited to 6 N.m (rated torque), In order to show the performances

 $\begin{array}{c|c} Table 2: Parameters of PMSM used in this study \\ \hline PMSM & Value (unit) \\ \hline P & 3 \\ Rs & 1.4 \Omega \\ L_t & 0.00066 \ H \\ L_q & 0.00058 \ H \\ J & 0.00176 \ kg \ m^{-2} \\ f & 0.00001 \ N.s/rad \\ \Phi_f & 0.15 \ Wb \\ \hline \end{array}$ 

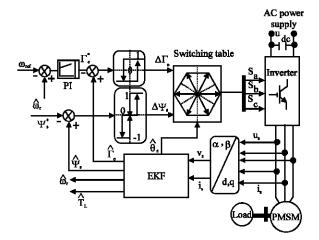


Fig. 4: Basic DTC for sensorless PMSM drives

and the robustness of the EKF algorithm, we simulated different cases, which are presented thereafter.

Comparison in the speed regulation level: Figure 5a presents the actual, estimated speeds  $\omega^r$  and  $\hat{\omega}_r$ , respectively. The estimated speed follows the real speed. Figure 5b presents the actual load torque and estimated load torque. The estimated torque follows accurately the load torque variations in static mode while in dynamic mode it presents a light shift due to the estimator reaction. Figure 6 presents the speed zoom with and without torque compensation. Figure 7 presents the estimated flux and the electromagnetic load torque. Figure 8 presents the speed variation from 100 rad sec<sup>-1</sup> to 150 rad sec<sup>-1</sup> with and without load torque compensation. Figure 9 presents Motor speed zoom with load torque compensation and without load torque compensation. Figure 10 shows the rotor Position. Figure 11 presents the actual load torque and estimated load torque.

**Inversion of the speed:** To test the robustness of the system, we applied a changing of the speed reference from 100 rad  $\sec^{-1}$  to -100 rad  $\sec^{-1}$  at t=0.5s. Figure 12 presents the actual  $\omega^r$  and estimated speed  $\hat{\omega}_r$ . The estimation algorithm is robust because the variation of the speed is important and the estimated speed follows the real speed when the motor starts and at the moment of speed inversion.

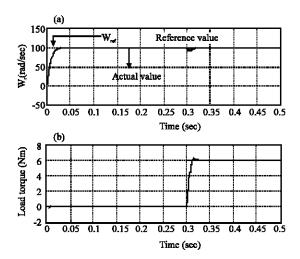


Fig. 5: (a) Actual speed and estimated speed, (b) Actual load torque (dashed line) and estimated load torque (solid line)

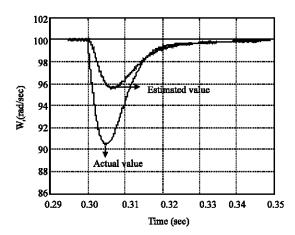


Fig. 6: Zoom of speed with and without torque compensation

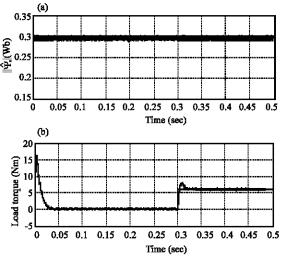


Fig. 7: Estimated stator flux magnitude and load torque

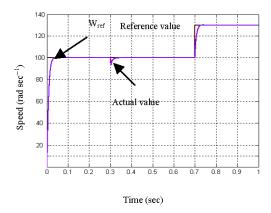


Fig. 8: Motor speed: with and without load torque compensation

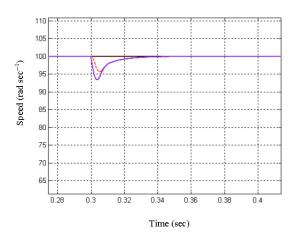


Fig. 9: Motor speed: with and without load torque compensation (zoom)

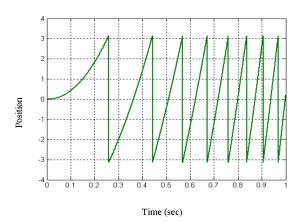


Fig. 10: Evolution of the rotor position

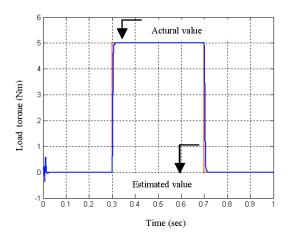


Fig. 11: Actual load torque and estimated load torque

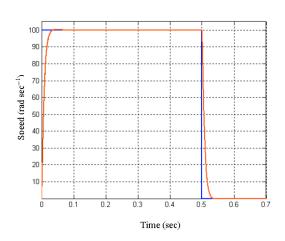


Fig. 12: Motor speed: with and without load torque compensation

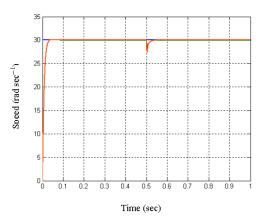


Fig. 13: Low speed: with and without load torque compensation

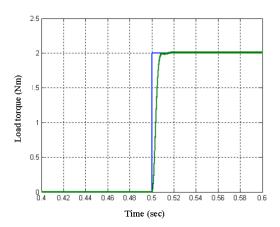


Fig. 14: Actual load torque and estimated load toreq

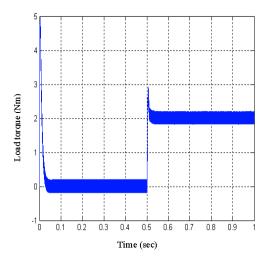


Fig. 15: Evolution of load torque

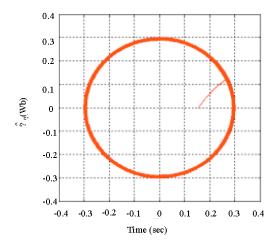


Fig. 16: Trajectory of the estimatyed stator flux components

**Operation at low speed:** The robustness of the proposed sensorless scheme has been tested at low speeds. Figure 13 shows the simulation waveforms when the speed is reduced at 30 (rad sec<sup>-1</sup>). The estimation algorithm ensures a high robustness since the estimated speed follows the real speed. Figure 14 presents actual load torque and estimated load torque. Figure 15 presents load torque where the estimated torque follows with a good precision the load torque variations. Figure 16 presents trajectory of the estimated stator flux components.

## CONCLUSION

In this study, the speed, rotor position and load torque estimation are presented using the EKF. The estimation of these states allows a Direct Torque Control (DTC) sensorless drive with high performance to be realized. Computer simulations have been carried out in order to evaluate the effectiveness of the proposed controller. The results prove that accurate tracking performance of the PMSM has been achieved at low speeds as well as high speeds. Moreover, this scheme is robust against the parameters variation and eliminates the influence of modeling and measurement noises. The filtering action of EKF improves the system performance, especially at low speeds. Simulation results reveal that the speed tracking are good and error convergence is guaranteed.

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