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Application of Linear Quadratic Regulator for UPS Systems

[1]Rabhi Boualaga, [1]Benaissa Amar and [1]Moussi Ammar and [2]M.A. Ayad
[1]MSE Laboratory, Department of Electrical Engineering, University of Biskra, Biskra 07000, Algeria
[2]ST Laboratory, University of Belfort-Montbéliard, Belfort, Cedex, 90010, France

Abstract: In this study an adaptive linear quadratic regulator for Uninterruptible Power Supplies (UPS) is proposed. In the controller design, the gains are determined by minimizing a cost function, which reduces the traching error and smoothes the control signal. A recursive least square estimator identifies the parameters model at different load conditions. Then the linear quadratic controller gains are adapted periodically. The output voltage is the only state variable measured.The other state variables are obtained by filtering. Simulation results show that the proposed strategy offers good performances for either linear and non-linear loads with low Total Harmonic Distortions (THD) even at low frequencies making it very useful for high power applications.

Key words: Uninterruptible power supplies, adaptive control, linear quadratic regulator, parameter estimation, simulation

INTRODUCTION

The ultimate goal of Uninterruptible Power Supplies (UPS)system is to supply constant amplitude sinusoidal voltage and constant frequency to load without any interruption in case of a main a power faillure. The quality of the UPS output voltage is defined by the Total Harmonic Distortion (THD). The most common UPS configuration consists of a battery bank and a static rectifier-inverter-filter that produce a low total harmonic distortion sinusoidal output voltage that supplies the critical load. The system performance is usually measured in term of transient response due to sudden changes in the load, waveform distortion with linear and nonlinear loads as well as efficiency (IEEE, 1995).

With the cost reduction of microcontrollers and Digital Signal Processors (DSP), the use of digital control technique in power converter has increased. However, high power converters are usually operated at low switching frequencies in order to reduce switching losses. Therefore, advanced control strategies are required to overcome these complications (Montagner and Carati, 2000; Karam and Mehdi, 2003).

To design the closed loop control, the model of the system has an important task in the conception of the controller. Some linear models for single phase PWM inverter system have been reported in literature (Kawamura and Yokoyama, 1991). The output voltage and its derivative, that is proportional to the capacitor current, can be used as the state variables, as well as the output voltage and the inductor current. However, modelling errors and unmodelled dynamics are quite common. They may be a result of simplifications on the model, which can degrade the performance of the system (Montagner and Carati, 2000).

Many discrete time controllers have been reported in the literature to control a single phase inverter for use in UPS. Among them guarantee a fast response for a load disturbance but have a high THD to nonlinear load, do not deal properly with parameters variation and functioned at high frequencies (Haneyoshi and Kawamara, 1988).

In this study an adaptive adaptive linear quadratic regulator for single-phase UPS application is proposed. The regulator is a useful tool in modern optimal control design. For the proposed controller, an recursive least square estimator identifies the plant parameters which are used to compute the regulator gains periodically. The quadratic cost function parameter is chosen in order to reduce the energy of the control signal. Only the output voltage can be measured and the inductor current is not measurable. As a result, an observer is used to estimate the inductor current. Using a suitable filter the effect of disturbances on the response of the system will be decreased. The simulations have been driven using SIMULINK.

Description of the plant: The single-phase PWM inverter is shown in Fig. 1, the LC filter and the resistive load R are considered to be the plant of the system.

Corresponding Author: Rabhi Boualaga, MSE Laboratory, Department of Electrical Engineering, University of Biskra, Biskra 07000, Algeria

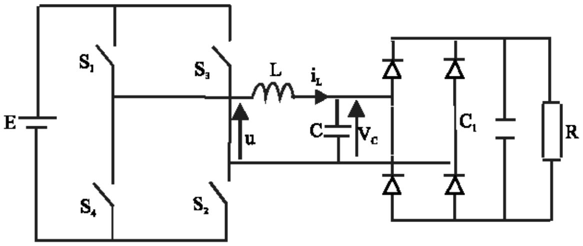

Fig. 1: Inverter, filter and load

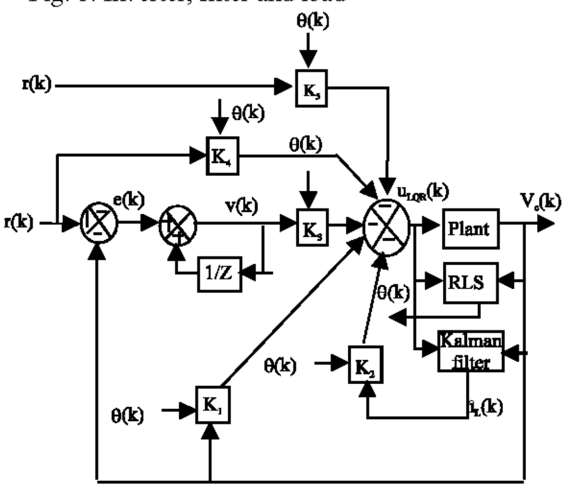

Fig. 2: Block diagram of the control system

The inverter is controlled by the unipolar PWM. The power switches are turned on and off at the carrier frequency.

The plant can be modelled by the state space variable v_C and i_L :

$$\begin{bmatrix} \dot{v}_c \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u, y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} \quad (1)$$

or

$$\dot{x} = Ax + Bu, y = Cx \quad (2)$$

Then, a discrete time model of the plant and sample time is given by:

$$x(k+1) = A_d x(k) + B_d u(k), y(k) = C_d x(k) \quad (3)$$

Where

$$x(k) = \begin{bmatrix} v_c(k) & \hat{i}_L(k) \end{bmatrix}^T$$

$$A_d = I + T_S A$$

$$B_d = TsB \quad (4)$$

Linear quadratic regulator: The adaptive linear quadratic regulator controller has the objective of tracking the discrete sinusoidal r(k) reference in each sample instant.

The system output y(k) is the capacitor voltage in the discrete form $v_c(k)$. The state variables used in the (LQR) are the measured output voltage $v_c(k)$, the estimated inductor current $\hat{i}_L(k)$ integrated tracking error v(k); all with a feedback action and the discrete reference r(k) and its derivative $\dot{r}(k)$ with a feed forward action. Each state variable has weighting Ki tuned according to $\theta(k)$, which contains the plant parameters identified by the RLS estimator. The control system shown in Fig. 2, is therefore proposed.

Then, in the proposed system, the state vector z(k) is defined as:

$$z(k) = \begin{bmatrix} v_c(k) & \hat{i}_L(k) & v(k) & r(k) & \dot{r}(k) \end{bmatrix}^T \quad (5)$$

and the LQR control signal is given by

$$u_{LQR}(k) = -Kz(k) \quad (6)$$

To design the optimal gainsK_1, K_2,..., K_5, the system must be represented in the form:

$$z(k+1) = G\,z(k) + Hu_{LQR}(k) \quad (7)$$

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Where each state variable is calculated by a difference equation. The two first variables of vector z(k) are obtained by (3). The signal v(k) is:

$$v(k+1) = e(k+1) + v(k) \quad (8)$$

Where the error is given by:

$$e(k) = r(k) - y(k) \quad (9)$$

From (3), (8) and (9) results the difference equation for

$$v(k+1) = v(k) + r(k) + T_S \dot{r}(k) - C_d A_d x(k) - C_d B_d u_{LQR}(k) \quad (10)$$

The continuous time reference variables are:

$$\begin{bmatrix} \dot{r}(t) \\ \ddot{r}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} r(t) \\ \dot{r}(t) \end{bmatrix}, \dot{r} = Rr \quad (11)$$

This system generates a sinusoidal reference when started with the values r(0)=0 and $\dot{a}(0) = wV_P$, where V_P is the sine wave amplitude and w is the angular frequency.

In the discrete form, using a sample period T_S, the subsystem (11) is given by:

$$n(k+1) = R_d n(k) \quad (12)$$

Where

$$n(k) = [r(k) \quad \dot{r}(k)] \quad (13)$$

$$R_d = I + T_S R \quad (14)$$

Then, using the state Eq. 3, 10 and 12, the closed loop system representation becomes:

$$\begin{bmatrix} x(k+1) \\ v(k+1) \\ n(k+1) \end{bmatrix} = \begin{bmatrix} A_d & 0 & 0 \\ -C_d A_d & 1 & C_d R_d \\ 0 & 0 & R_d \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \\ n(k) \end{bmatrix} + \begin{bmatrix} B_d \\ -C_d B_d \\ 0 \end{bmatrix} u_{LQR}(k)$$

$$y(k) = [C_d \quad 0 \quad 0][x(k) \quad v(k) \quad n(k)]^T \quad (15)$$

The optimal gains of the control law (6) are those that minimize the following cost function:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} \left\{ \begin{aligned} & z^T(k) Q z(k) + \\ & u^T(k) R_u u(k) \end{aligned} \right\} \quad (16)$$

Where Q and R_u in (16) are chosen as positive definite matrixes that set the weighting of each state and of the control signal.

The K gains can be obtained through the evaluating the Riccati Equations (Ogata, 1987). as follows:

$$S(k) = G^T S(k+1) G + Q - \left[H^T S(k+1) G\right]^T \times \left[R_u + H^T S(k+1) H\right]^{-1} \left[H^T S(k+1) G\right] \quad (17)$$

$$K(k) = R_u^{-1} H^T \left(G^T\right)^{-1} \left(S(k) - Q\right) \quad (18)$$

A good flexibility in the design of the controller is provided by the selection of Q and R_u matrixes.

Recursive Least Squares (RLS) estimator: To estimate the plant parameters when the load conditions are variable, a RLS algorithm is used (Astrom and Wittenmark, 1995). The discrete plant model with a zero order hold is given by:

$$\frac{y(z)}{u(z)} = \frac{\theta_3}{z^2 + \theta_1 z + \theta_2} \quad (19)$$

The difference equation of the estimated output is:

$$y(k) = -\theta_1 y(k-1) - \theta_2 y(k-2) + \theta_3 u(k-2) \quad (20)$$

or

$$\hat{y}(k) = \theta^T(k) \Psi(k-1) \quad (21)$$

Where

$$\theta(k) = [\theta_1 \quad \theta_2 \quad \theta_3] \quad (22)$$

and

$$\Psi(k) = [-y(k-1) \quad -y(k-2) \quad u(k-2)] \quad (23)$$

The RLS gains are calculated using:

$$L(k) = \frac{p(k-1)\Psi(k)}{1 + \Psi^T k) p(k-1) \Psi(k)} \quad (24)$$

The RLS covariance matrix is given by:

$$p(k) = p(k-1) - \frac{p(k-1)\Psi(k)\Psi^T(k) p(k-1)}{1 + \Psi^T(k) p(k-1) \Psi(k)} \quad (25)$$

and the plant parameters (are recursively obtained by:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k)\left[y(k) - \Psi^T \hat{\theta}(k-1) \right] \quad (26)$$

Where:

$$\hat{A}_d = \begin{bmatrix} 0 & -\hat{\theta}_2 \\ 1 & -\hat{\theta}_1 \end{bmatrix}, \hat{B}_d = \begin{bmatrix} \hat{\theta}_3 \\ 0 \end{bmatrix}, C_d = [0 \quad 1] \quad (27)$$

Then, it is possible to identify the plant parameters to a range of different loads and to substitute the matrixes in (27) into the system in (15) to proceed with the LQR gains design in real time.

Kalman filter: Since only the output voltage is measured, a Kalman filter (Welch and Bishop, 2003). is used to estimate the inductor current state.

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) + w(k) \\ y(k) &= C_d x(k) + v(k) \end{aligned} \quad (28)$$

The random variables w(k) and w(k) represent the process and measurement noise, respectively. They are assumed to be independent of each other and with normal probability distributions such that:

$$\begin{aligned} E\left[w(k)^T \quad w(k)\right] &= R_W \rangle 0 \\ E\left[v(k)^T \quad v(k)\right] &= R_V \rangle 0 \\ E\left[w(k)^T \quad v(k)\right] &= 0 \end{aligned} \quad (29)$$

In practice, the process noise covariance and measurement noise covariance matrices might change with each time step or measurement. However, here, it is assumed that they are presented below (Welch and Bishop, 2003).
The Kalman gains are given by:

$$K_G(k) = \left(M(k) C_d^{\,T}\right)\left(C_d M(k) C_d^{\,T} + R_v\right)^{-1} \quad (30)$$

and the estimated variable, the inductor current, is

$$i_L = \hat{x}_2(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{x}(k) \quad (31)$$

The following recursive equations are used:

$$P_K(k) = M(k) - K_G(k) C_d M(k) \quad (32)$$

and

$$M(k) = \left(A_d P_K(k) A_d^{\,T}\right) + \left(B_d R_W B_d^{\,T}\right) \quad (33)$$

After each time and measurement update pair, the process is repeated with the previous posterior estimates used to project or predict the new a priori estimates.

RESULTS AND DISCUSSION

The simulation bloc diagram is shown in Fig. 3. The inverter system controlled by linear quadratic

Table 1: System parameters

DC input voltage	E = 400V
Reference voltage	Vref = 320 V (peak), 60 Hz
Sample time	Ts = 1/I8000s
States weightings	Q = diag [50 100 150 1 1]
Control weighting	Ru = 100
For linear load:	
Filter inductance	L = 5.3 mH
Filter capacitance	C = 80 µF
Linear load	R = 6Ω
LQR gains	K = [8.0177 36.0875 -1.0127 -10.0251 -0.0031]
For non linear load:	
Non linear load	Diode bridge rectifiers with RC_1(R = 56Ω, C_1 = 1000 µF)
Filter inductance	L = 0.5 mH
Filter capacitance	C = 800 µF
LQR gains	K = [7.90556 3.5968 -0.9990 -9.9400 -0.0031]

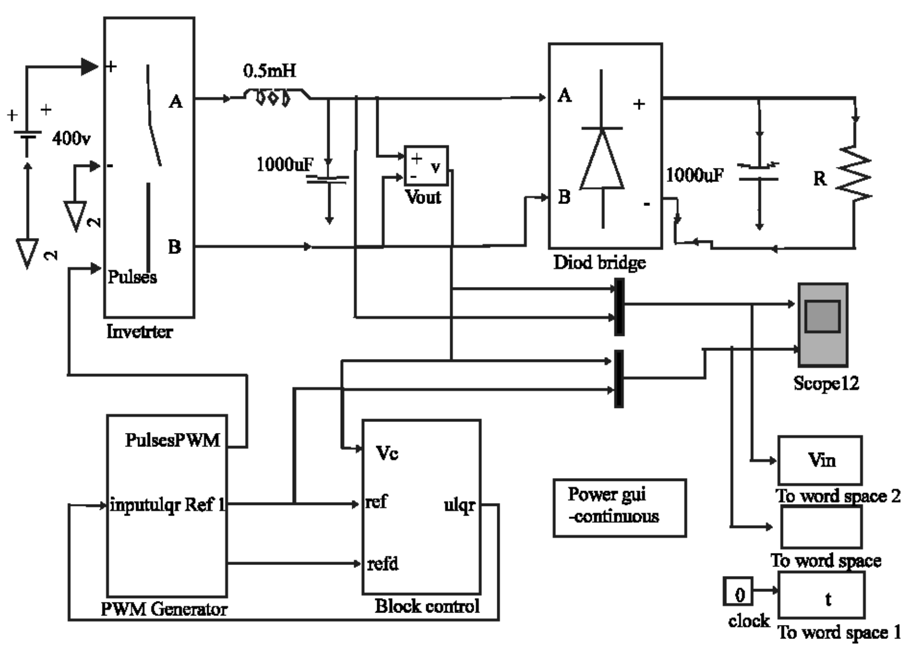

Fig. 3: System simulink bloc diagram

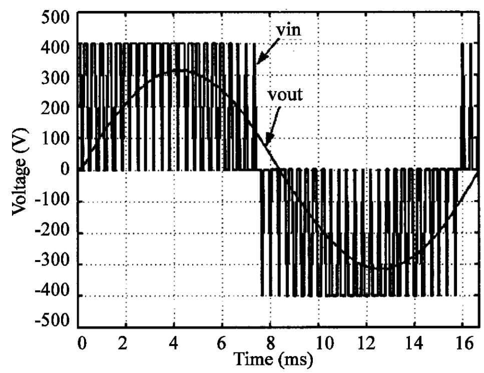

Fig. 4: Input and output voltage for a linear load

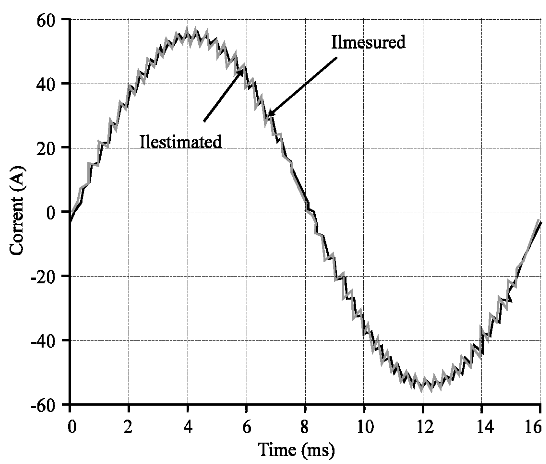

Fig. 5: Measured and estimated inductor current for a linear load

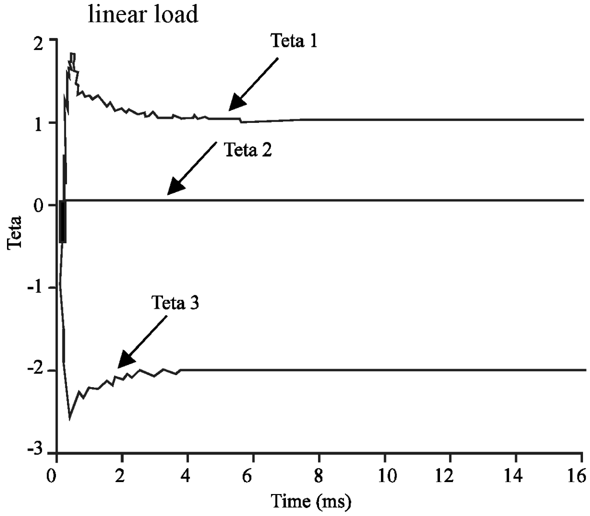

Fig. 6: The estimated parameters for a linear load

voltage v_c under linear and nonlinear loads. The plant and controller parameters, algorithm constants and other system specifications are shown in Table 1.

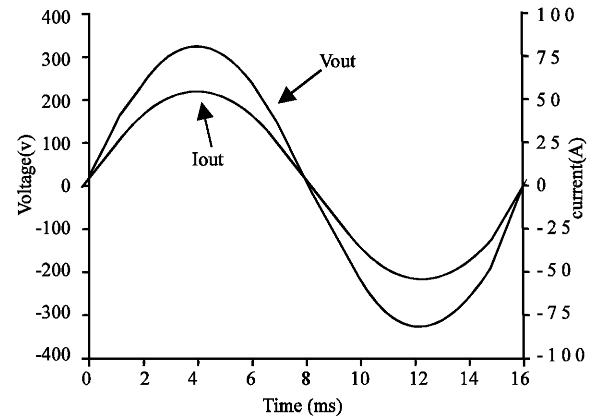

Fig.7: Output voltage and current for a linear load

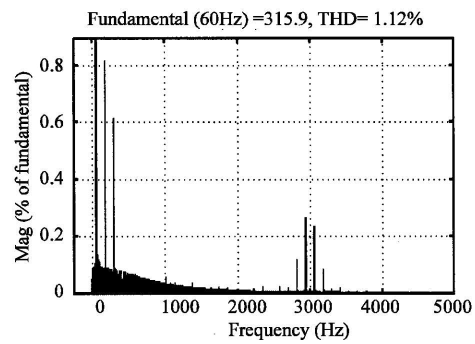

Fig. 8: Spectral analysis of the output voltage for a linear load.

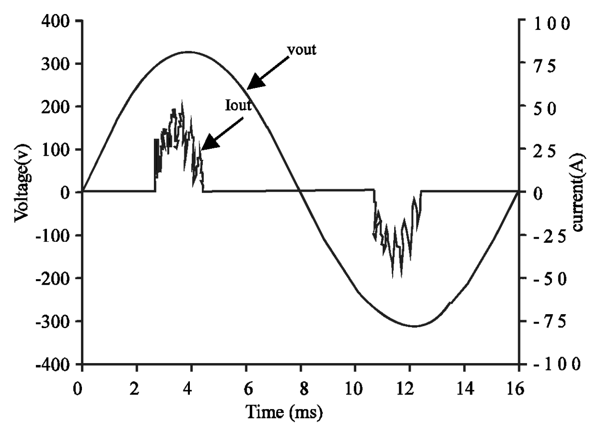

Fig. 9: Output voltage and current for a non linear load.

The Input voltage Vin and output voltage for the linear load are shown in Fig. 4. The measured and estimated inductor currents are shown in Fig. 5. For this study, the estimated parameters are shown in Fig. 6. The output voltage and current with the linear load ($R = 6\Omega$) and the gains K shown in Table I are shown in Fig. 7. The spectrum of the output voltage for this case is shown in Fig. 8.

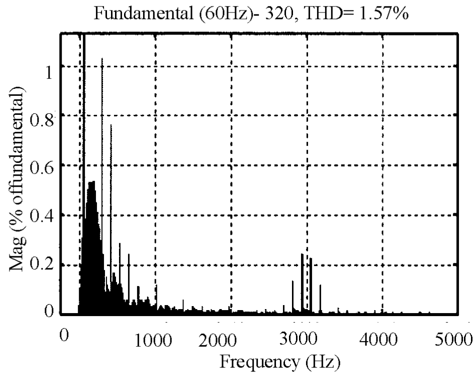

Fig. 10: Spectral analysis of the output voltage for a non linear load

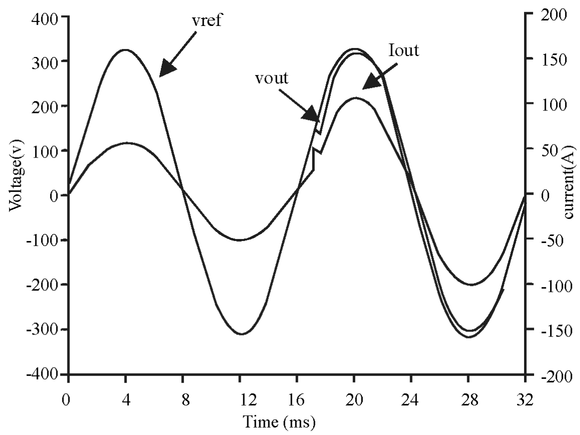

Fig. 11: Reference voltage, output voltage and current with linear load disturbance (from R = 6Ω to R = 3Ω)

The output voltage and current with the non linear load are shown in Fig. 9. For this case the spectrum of the output voltage is shown in Fig. 10.

The efficiency of the LQR regulator is well illustrated in Fig.11. it is shown that the output voltage follows efficiently the reverence voltage in case of linear load disturbance.

CONCLUSION

A Linear Quadratic Regulator was successfully developed for a single phase UPS application. The linear quadratic regulator gains are calculated by minimizing a cost function which can be changed by the designer by modification of the weighting factors. Therefore, it is possible to reduce the control efforts in tracking the sinusoidal reference. The RLS estimator identifies the plant parameters which are used to compute LQR gains periodically.

The discrete control law has shown good performances to linear and nonlinear loads when operated at low switching frequency. Theses characteristics make this scheme suitable to be used in high power applications as well as to be implemented through a low cost micro controller.

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