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Backstepping Control Design for Position and Speed Tracking of DC Motors

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Abstract: The study deals with the tracking control problem of DC motors using backstepping design. This approach is based on the use of the non linearites in the DC motor model with both electrical and mechanical dynamics and finding a direct relationship between the motor output and input quantities without affecting the speed regulation. The proposed control scheme is not only to stabilize the DC motor, but also to drive the speed tracking error to converge to zero asymptotically. System robustness and asymptotic position tracking performance are shown through simulation results.

Key words: Backstepping control, DC motors, lyapunov stability, nonlinear systems

INTRODUCTION

DC drive systems are often used in industrial applications such as robotics where a wide range of speed or position control is required. In other applications dc motors are used to follow a predetermined speed or position track under variable load. Various nonlinear control and adaptive control methods for nonlinear system have been proposed. Among many others, those control methods include: input-output linearization with or without adaptive control[1,2], feedback linearization and pseudo linearization[3], adaptive control[4-6].

In the past decade, research on backstepping control has increased[7-11]. The backstepping control is a systematic and recursive design methodology for nonlinear feedback control. Applying those design methods, control objectives such as position, velocity, can be achieved. Obviously, the control approach mentioned above leads to nonlinear controllers that are usually very complicated, and implementing this controller requires high performances microprocessors. Furthermore, this design method requires the access to the whole system dynamics, including both the 'inner' electrical subsystem and the "outer" mechanical subsystem.

This study investigates the application of the backstepping control technique for tracking control of dc motors. The objective is to make the motor output y(t) track a desired trajectory y_{ref}. This tracking can be achieved through a backstepping algorithm by finding a direct relation between the motor output y(t) and its control input u(t). The theoretical bases of the proposed control technique are derived in detail and some simulation results are provided to demonstrate the effectiveness of the proposed schemes.

Backstepping theory: The control objective is to design a robust controller for the output y(t) of the system to tack the output y_{ref}, of the reference model asymptotically. Assume that not only y_{ref}, but also its first two derivatives $\dot{y}_{ref}$ and $\ddot{y}_{ref}$ are all bounded functions of time. The diagram block of this backstepping control is shown in Fig. 1.

The backstepping design to achieve the position tracking objective is described step-by-step as follows. Consider a drive system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u + \varphi(x_1, x_2)\theta \\ y = x_1 \end{cases} \tag{1}$$

Where u : Control input
θ : System parameters
y : Output state.
x : Variable state.

Step 1: For the position tracking objective, define the tracking error as

$$z_1 = y - y_{ref} \tag{2}$$

And its derivative as:

$$z_1 = y - y_{ref} \tag{3}$$

The x_2 can be viewed as a virtual control in above equation. Define the following stabilizing function

$$\alpha_1 = -c_1 z_1 \tag{4}$$

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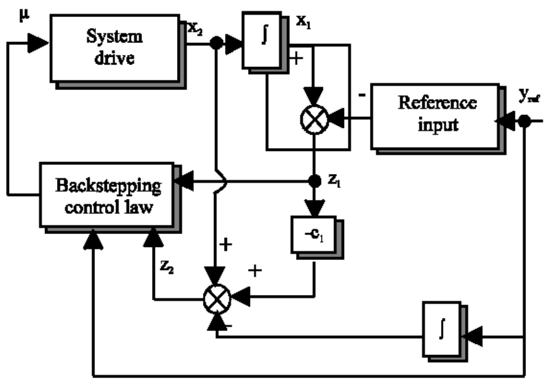

Fig. 1: Backstepping control system

Where, c_1 is a positive constant. So, the second regulated variable is chosen as:

$$z_2 = x_2 - \alpha_1 - \dot{y}_{ref} \tag{5}$$

The first Lyapunov function is chosen as:

$$V_1 = \frac{1}{2} z_1^2 \tag{6}$$

Then the derivative of V_1 is

$$\dot{V}_1 = z_1 z_2 - c_1 z_1^2 \tag{7}$$

Step 2: Hence, the derivative of the second regulated variable is calculated as

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 - \ddot{y}_{ref} \tag{8}$$

To design the controller, add terms concerning z_2 to V_1 to form the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{9}$$

Using Eq. 7 and 9, the derivative of V_2 can be derived as follows:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2 \dot{z}_2 \\ &= -c_1 z_1^2 + z_2 \left(z_1 + \dot{z}_2 \right) \end{aligned} \tag{10}$$

According to (10), the control law u is designed as follows:

$$u = -c_2 z_2 - z_1 - \varphi(x_1, x_2)\theta - c_1 \dot{z}_1 + \ddot{y}_{ref} \tag{11}$$

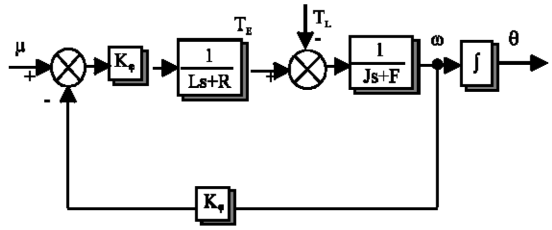

Fig. 2: DC Motor system dynamic model

Where, c_2 is a positive constant. Substituting Eq. 11 into Eq. 10, the following equation can be obtained

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \leq 0 \tag{12}$$

So, the backstepping control is asymptotically stabilizing.

Dc motor model: A dynamic model of the dc motor is used for designing the tracking controller; a third-order dc motor into state-space form which includes both the electrical and mechanical dynamics is given as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{F}{J} x_2 + \frac{1}{J} x_3 - \frac{1}{J} T_L \\ \dot{x}_3 = -\frac{K_\varphi^2}{L} x_2 - \frac{R}{L} x_3 + \frac{K_\phi}{L} u \end{cases} \tag{13}$$

Where x_i is the motor states variables, $[x_1\ x_2\ x_3] = [\theta\ \omega\ T_E]^T$ is the input voltage.

θ	Rotor position (radians)
ω	Rotor speed (rad/s)
T_E	Electromagnetic torque (N.m)
T_L	Load torque (N.m)
u	Terminal voltage (V)
R, L	Armature resistance (Ω), inductance (H)
J	Rotor inertia (Kg-m^2)
F	Coefficient of viscous friction.
K_φ	Torque constant (N.m/A).

A block diagram of the DC motor system is shown in Fig. 2. A separately excited dc motor with constant field current is considered in this study, where K_φ is considered constant.

Backstepping design steps: The model developed in the preceding section is now used to design the backstepping algorithm for rotor position and speed tracking of a separately excited dc motor, to achieve the stability and position tracking objectives.

Step 1: First of all, we regard the velocity x_2 as the control variables. The first regulated variable is the position tracking error signal.

$$z_1 = x_1 - y_{r1} \tag{14}$$

Where y_{1r} is the reference signal of angular velocity, the derivative of (14) is computed as

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{y}_{r1} \\ &= x_2 - \dot{y}_{r1} \end{aligned} \tag{15}$$

If x_2 is the control, the first Lyapunov candidate V_1 is chosen as:

$$V_1 = \frac{1}{2} z_1^2 \tag{16}$$

So the derivative of (16) is computed as:

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 \\ &= z_1 (x_2 - \dot{y}_{r1}) \end{aligned} \tag{17}$$

At this point, the desired reference velocity can be selected by:

$$y_{r2} = -c_1 z_1 + \dot{y}_{r1} \tag{18}$$

Where c_1 is a positive design constant, therefore, (17) can be rewritten as:

$$\dot{V}_1 = -c_1 z_1^2 < 0 \tag{19}$$

The control y_{r2} in (18) is asymptotically stabilizing which indicates the desired velocity trajectory for position tracking. So the next step is the design to make the velocity behave according to (18).

Step 2: The purpose of this control design is to achieve the reference torque, so the second regulated variable is chosen as

$$\begin{aligned} z_2 &= x_2 - y_{r2} \\ &= x_2 + c_1 z_1 - \dot{y}_{r1} \end{aligned} \tag{20}$$

With this definition, the dynamical equation of the error signal z_1 can be expressed as:

$$\dot{z}_1 = -c_1 z_1 + z_2 \tag{21}$$

Hence, the derivative of (20) is calculated as:

$$\dot{z}_2 = c_1 \left[z_2 - c_1 z_1 \right] + \frac{F}{J} x_2 - \frac{1}{J} x_3 + \frac{1}{J} T_L - \ddot{y}_{r1} \tag{22}$$

Again, if all the parameters are known, the second Lyapunov candidate is:

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \tag{23}$$

To derive a stabilizing control certainly V_2 is obtained from V_1 with an added term of z_2.

$$\begin{aligned} \dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2 = \; & z_1 \left[z_2 - c_1 z_1 \right] + \\ & z_2 \left[-c_1^2 z_1 + c_1 z_2 - \ddot{y}_{r1} + \frac{F}{J} x_2 - \frac{1}{J} x_3 + \frac{T_L}{J} \right] \end{aligned} \tag{24}$$

In order to make the derivative of the complete Lyapunov function (24) be negative define, the torque reference is chosen as follows:

$$y_{r3} = J \left[(1 - c_1^2) z_1 + (c_1 + c_2) z_2 - \ddot{y}_{r1} \right] - F x_2 - T_L \tag{25}$$

Then, we get:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 \le 0 \tag{26}$$

Step 3: Furthermore, the final error signal is defined as

$$z_3 = x_3 - y_{r3} \tag{27}$$

The derivative of the given error variable z_3 is computed as:

$$\begin{aligned} \dot{z}_3 = \; & J(z_2 - c_1 z_1)(1 + c_1 c_2) + x_2 (F(c_1 + c_2) + F^2 - a_1) \\ & + (c_1 + c_2) \ddot{y}_{r1} J - x_3 (F(1 + c_1 + c_2) - a_2) + J \dddot{y}_{r1} \\ & T_L (c_1 + c_2 + F) - J \dot{T}_L - bu \end{aligned} \tag{28}$$

Where $a_1 = \frac{K_\varphi^2}{L}$, $a_2 = \frac{R}{L}$, $b = \frac{K_\varphi}{L}$

With the selection of the complete Lyapunov function

$$V_3 = \frac{1}{2} z_3^2 \tag{29}$$

In order to make negative the derivative of the complete Lyapunov function (29), the torque control input is chosen as:

$$\dot{z}_3 = -c_3 z_3 \tag{30}$$

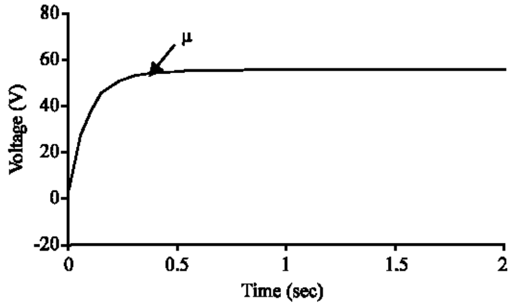

Fig. 3: Input command for step of rotor speed reference

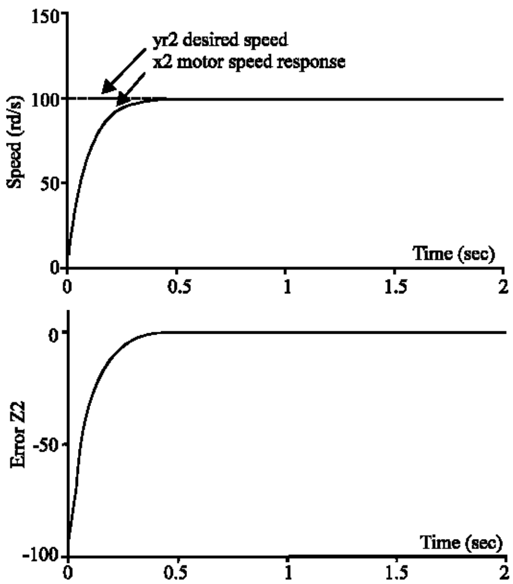

Fig. 4: Motor rotor speed response for step of rotor speed reference

Where, $c_3 > 0$ is a design constant, then, we get

$$\dot{V}_3 = z_3\dot{z}_3 = -c_3 z_3^2 \leq 0 \tag{31}$$

Therefore, substituting (28) to (31), we are able to obtain the control law as:

$$\begin{aligned} u = \frac{1}{b}\big[& J(z_2 - c_1 z_1)(1 + c_1 c_2) + x_2(F(c_1 + c_2) + F^2 - a_1). \\ & + (c_1 + c_2)\ddot{y}_{r1} J - x_3(F(1 + c_1 + c_2) - a_2) + J\ddot{y}_{r1} \\ & T_L(c_1 + c_2 + F) - J\dot{T}_L + c_3 z_3 \big] \end{aligned} \tag{32}$$

Clearly, $\dot{V}_1$, $\dot{V}_2$, $\dot{V}_3$ in (19), (26) and (31) are negatives, so it implies that the resulting closed loop system is asymptotically stable and hence, all the errors variables z_1, z_2 and z_3 will converge to zero asymptotically. As a result, from the definition of (14) and (20), the angular rotor position and the rotor speed will converge to the reference speed. The above equation will be used to show that the closed loops system (13) is asymptotically stable and the positive tracking objective is achieved.

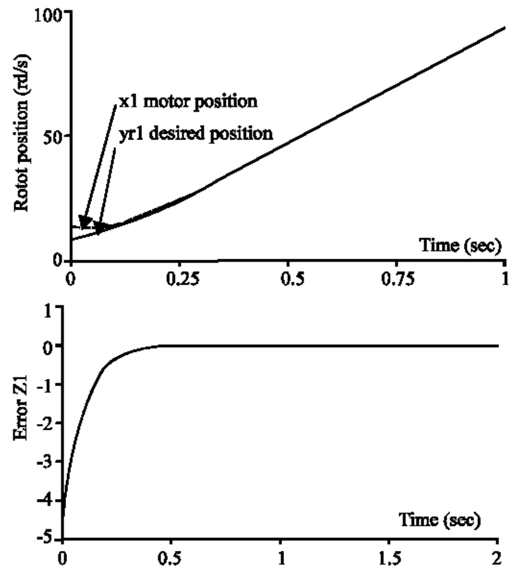

Fig. 5: Motor rotor position response for step of rotor speed reference

Simulation results: To verify the stability and asymptotic tracking performance of the above adaptive backstepping design, simulations are carried out in Matlab/Simulink. The effectiveness of the proposed tracking controller was evaluated for position and speed tracking of a separately excited DC motor model. The machine parameters according to the data acquired from an experiment platform taken from[12] are: J = 0.093 Kg.m^2, F = 0.008, K_ϕ = 0,55 Nm.A, R = 1Ω, L = 0,046H.

The parameters c_i used in the backstepping control are chosen as follows: c_1 = 1, c_2= 1, c_2 = 3 to satisfy convergence conditions. The speed reference signal is chosen in the following two studies.

Case 1: y_{ref}, which is a constant reference

Figure 3 shows the associated control input waveform of DC motor using backstepping control. Figure 4 shows the desired speed trajectory and rotor speed response. It is noticed from these figures that speed trajectory have been satisfactorily obtained from null error given by z_2

From the speed tracking simulation results, we can see that the rotor position and the desired one present a zero error given by z_1 presented in Fig. 5. The electromagnetic torque and load torque are presented in Fig. 6, it is clear that the electromagnetic torque is always forced to converge to the load torque with zero error. We note an improved stability of rotor position, rotor speed and electrical torque.

Case 2: y_{ref}, which is periodic reference signal.

The second rotor speed reference signal is a sinusoidal type. In this study, it indicates the tracking performance and control input by Fig. 7. The actual speed responses given by Fig. 8 can also track the reference speed in the beginning when the reference speed is a sinusoidal signal. The error between the actual speed and the reference speed indicate that the proposed backstepping control is effective.

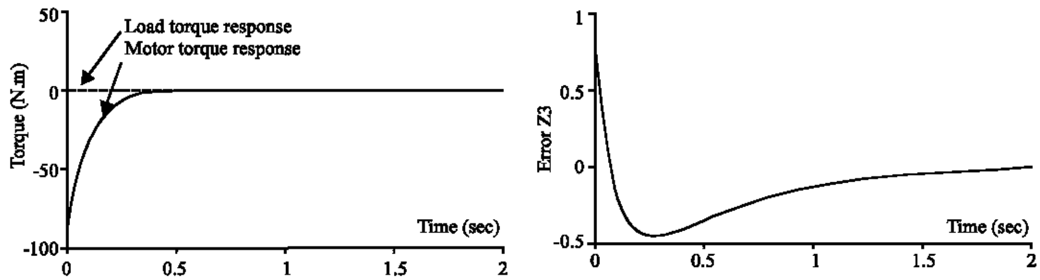

Fig. 6: Motor load torque and electrical torque for step of rotor speed reference

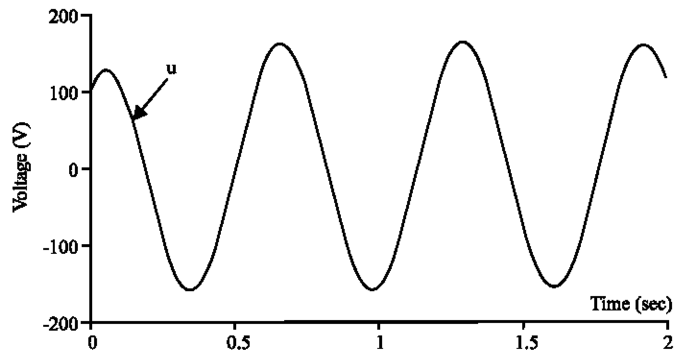

Fig. 7: Input command for sinusoidal rotor speed reference

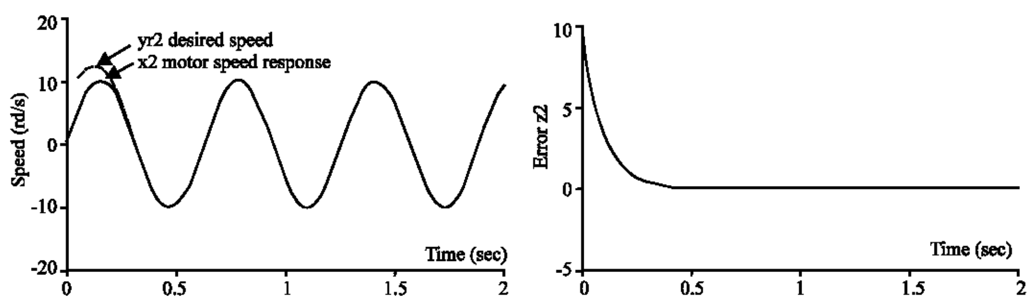

Fig. 8: Motor rotor speed response for sinusoidal rotor speed reference

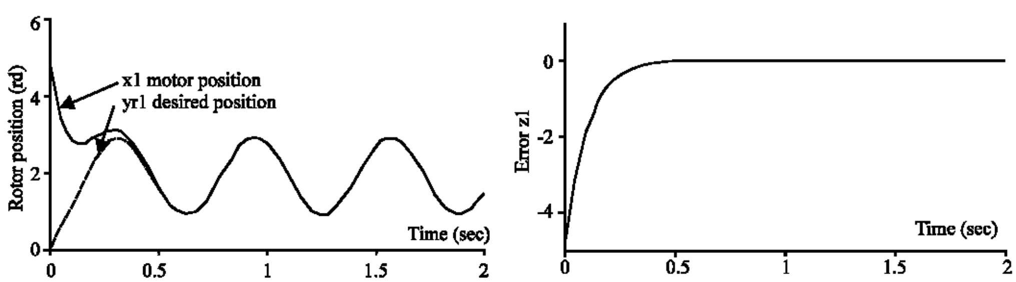

Fig. 9: Motor rotor position response for sinusoidal rotor speed reference

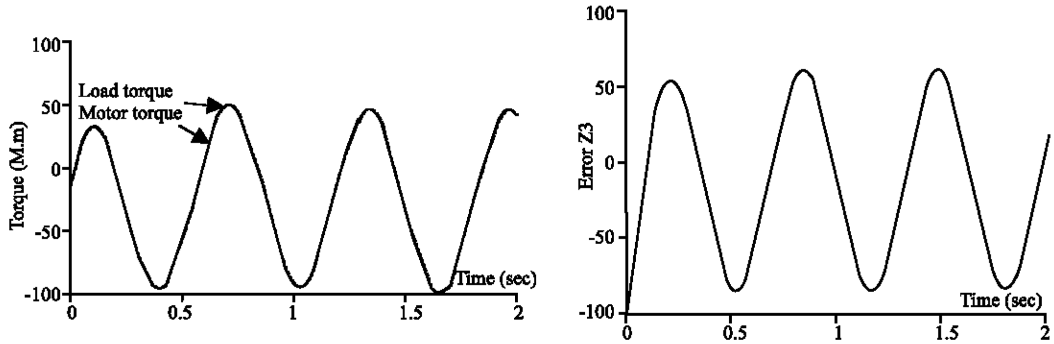

Fig. 10: Motor load torque and electrical torque for sinusoidal rotor speed reference

We can find also, that the rotor position and the torque errors given by Fig. 9 and 10 converge to zero rapidly. These references signals has sinusoidal forms, it can be seemed that the rotor position and the electrical torque can also track the references. From these simulation results, it is obvious that the proposed backstepping controller is quite successful and presents an excellent performance.

CONCLUSION

The study describes a nonlinear backstepping design scheme for the control of dc motor system to achieve the desired speed tracking control objective. The resulting closed-loop system is guaranteed to be asymptotically stable, and the speed tracking error and the position error are able to converge to zero asymptotically according to Lyapunov stability theorem. Step by step control design and stability analysis are given and the effectiveness of this design is demonstrated through computer simulations. In addition, the simulation results have clearly illustrated that the proposed nonlinear backstepping controllers are quite effective and efficient for dc machine. The asymptotic tracking performance and the effectiveness of the control were achieved.

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