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Performance Study of LMS and RLS Algorithms in the Identification of Nonlinear Volterra Systems

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Abstract: Nonlinear filters are currently used in many applications where the performance of linear filters is unacceptable. The quadratic Volterra filters are introduced as simplest models for the analysis and the identification of nonlinear systems. In this paper, a comparative study of the performances of the LMS and RLS algorithms in the identification of quadratic Volterra systems is presented. A simulation based on a test model is used to show the powerful feature of the RLS algorithm. Moreover, the convergence speed of each algorithm is evaluated according to the variation of the mean-square error criterion on linear and quadratic kernels of the Volterra filters. Also, the effect of noise on the detection feature of the LMS and RLS algorithms are considered in this study.

Key words: Nonlinear filters, volterra systems, LMS, RLS algorithms, identification

INTRODUCTION

The linear modeling is widely used in signal processing applications because of its implementation simplicity[1,2]. However, the are many situations in which linear filters perform poorly. For this reason, nonlinear filtering has been applied by many researchers to analyze and study various phenomena such as: EMG signal processing, characterization of semi-conductor components, etc[2]. From the various categories of nonlinear filters, we find the intensive use of Volterra filters to handle the small non linearities in the scientific literature[3]. Quadratic Volterra filters are the simplest polynomial filters which correspond to the first nonlinear term in the expansion. These filters, requiring a limited amount of knowledge of high-order statistics, lead to realizations of reasonable complexity. Two important features make quadratic Volterra filters very attractive. The first one is that the output of a Volterra filter depends linearly on the coefficients of the filter. This property permits an easy extension of linear adaptive algorithms to Volterra filters. The second characteristic results from representing the non linearity by means of multidimensional operators working on products of input samples. This last feature is largely used to describe the filter behavior in the frequency domain by means of a type of multidimensional convolution[3,4]. Thus the discrete Volterra filter can be considered as a mathematical description that extends the well-known linear approach for the analysis and synthesis of nonlinear systems[3].

In this study, a comparative study of the LMS and RLS adaptive algorithms applied to quadratic Volterra filters is presented. These algorithms are used to estimate the linear and quadratic Volterra kernels. Moreover, an investigation of the effect of additive noise on the usefulness limits of the LMS and RLS algorithms is considered.

The volterra series: In This study, the general Volterra theory is presented. In the Volterra series representation of a nonlinear system, the output y(n) of a discrete causal time-invariant nonlinear system is expressed as a function of the input sequence x(n).

$$y(n) = h_0 + \sum_{m_1=0}^{\infty} h_1(m_1)x(n-m_1) + \sum_{m_1=0}^{\infty}\sum_{m_2=m_1}^{\infty} h_2(m_1,m_2x)(n-m_1x)(n-m_2) + \dots + \sum_{m_1=0}^{\infty}\cdots\sum_{m_p=m_{p-1}}^{\infty} h_p(m_1,\dots,m_p)x(n-m_1)\dots x(n-m_p) \quad (1)$$

where $h_p(m_1,\dots,m_p)$ is the Volterra kernel of the p^{Th} order.

In[1,2,5,6], these kernels are supposed to be symmetric (i.e., $h_p(m_1,\dots,m_p)$ does not change for the p! possible permutations of $m_1,\dots, m_p$ indices).

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In the rest of this work, the 2nd order truncated Volterra series is used.

ADAPTIVE QUADRATIC VOLTERRA FILTERS

Figure 1 represents a quadratic Volterra model where the input x(n) and the output y(n) are related via a second order truncated Volterra series. E(n) is the measured error and y(n) the estimated output.

In this study, the LMS and RLS adaptation algorithms are used for the determination of $h_1(m_1)$ and $h_2(m_1,m_2)$ kernels. These two algorithms are presented in the following section.

$$y(n) = \sum_{m_1=0}^{\infty} h_1(m_1 x)(n-m_1) + \sum_{m_1=0}^{\infty}\sum_{m_2=m_1}^{\infty} h_2(m_1,m_2)x(n-m_1)x(n-m_2) = \hat{y}(n) + e(n) \quad (2)$$

The LMS algorithm: The LMS algorithm assumes input vectors uncorrelated over time with stationary input and output signals. This algorithm adapts the linear and quadratic Volterra kernels using the steepest descent algorithm which minimizes $e^2(n)$ at each time n. The adaptation equations for the quadratic Volterra filter are given by:

$$h_1(m_1,n+1) = h_1(m_1,n) - \frac{\mu_1}{2}\frac{\partial e^2(n)}{\partial h_1(m_1,n)} = h_1(m_1,n) + \mu_1\, e(n)\, x(n-m_1) \quad (3)$$

for linear kernels $h_1(m_1)$ and

$$h_2(m_1,m_2,n+1) = h_2(m_1,m_2,n) - \frac{\mu_2}{2}\frac{\partial e^2(n)}{\partial h_2(m_1,m_2,n)} = h_2(m_1,m_2,n) + \mu_2 e(n)x(n-m_1)x(n-m_2) \quad (4)$$

for quadratic kernels $h_2(m_1,m_2)$.

$\mu 1$ et $\mu 2$ are positive constants that control the convergence speed and filter stability. It is shown in the reference[8] that LMS algorithm converges quadratically if the constants μ_1 and μ_2 satisfy the condition

$$0 < \mu_1, \mu_2 < \frac{2}{\lambda_{max}} \quad (5)$$

with λ_{max} the largest eigenvalue of the autocorrelation matrix of the input x(n),

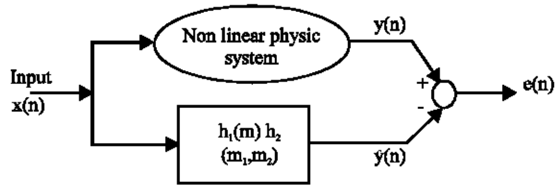

Fig. 1: Quadratic volterra model[7]

or the condition

$$0 < \mu_1, \mu_2 < \frac{2}{\sum_{k=0}^{m} \lambda_k} \quad (6)$$

with λ_k the eigenvalues of the autocorrelation matrix R_x.

The choice of $\mu 1$ and $\mu 2$ given by the expressions of Eq. 7[6,9]. These values are adopted in our simulation part, with $0<\alpha_1$, $\alpha_2<2$.

$$\mu_1 = \frac{\alpha_1}{\| X_1 \|^2} \quad \text{et} \quad \mu_2 = \frac{\alpha_2}{\| X_2 \|^2} \quad (7)$$

The RLS algorithm: The direct evaluation of the RLS solution for both linear and quadratic coefficients of the adaptive Volterra filter by minimizing the cost function:

$$J(n) = \sum_{k=0}^{n} \lambda^{n-k} [\ y(k) - H(n)X(k)\ ]^2 \quad (8)$$

at each time n, requires $O(N^6)$ multiplications per iteration[2,5], by using the matrix inversion lemma given by Schur, this computation complexity can be reduced to $O(N^4)$[5] multiplications per iteration which is still very large compared to the quantity $O(N^2)$[2] of the LMS solution.

The RLS solution using Schur algorithm[10] is given by:

$$H(n) = H(n) + K(n)\left[d(n) - H^T(n-1)X(n)\right] \quad (9)$$

where

$$K(n) = \frac{\lambda^{-1} C^{-1}(n-1)\, X(n)}{1 + \lambda^{-1} X^T(n)\, C^{-1}(n-1)\, X(n)}$$

and

$$C^{-1}(n) = \lambda^{-1}C^{-1}(n-1) - \lambda^{-1}K(n)X^T(n)C^{-1}(n-1)$$

with λ a weighting factor which is a positive constant less than (often very near) unity.

A simulation of the LMS and RLS algorithms is presented in the following section. This simulation permits to compare the performances of this two algorithms in the representation of the nonlinear systems with the quadratic Volterra model.

RESULTS AND DISCUSSION

In this study, we present a comparative study of the performances of both LMS and RLS algorithms applied to the identification of quadratic Volterra systems.

In order to compare the performances of LMS and RLS algorithms, we use a quadratic Volterra model. this model, corrupted by additive noise, is defined with a second order truncated Volterra series of the linear coefficients:

$$H_1 = \left[h_1(1), h_1(2)\right] = \left[0.65, -0.35\right]$$

and quadratic coefficients:

$$H_2 = \left[h_2(1,1), h_2(1,2), h_2(2,2)\right] = \left[0.5, -0.5, 0.25\right]$$

The simulation conditions are: a gaussian input, an output of a unity variance and input signal-to-noise ratio equal to 30dB. The variance of the gaussian additive noise is fixed to $2.14 * 10^{-4}$.

Figure 2 shows the learning curves of the Volterra coefficients estimated by LMS and RLS algorithms. Figure 3 shows the curves of the quadratic error on the estimation of linear Volterra coefficients by LMS and RLS algorithms. This error is given by Mathews and Lee[1,5]:

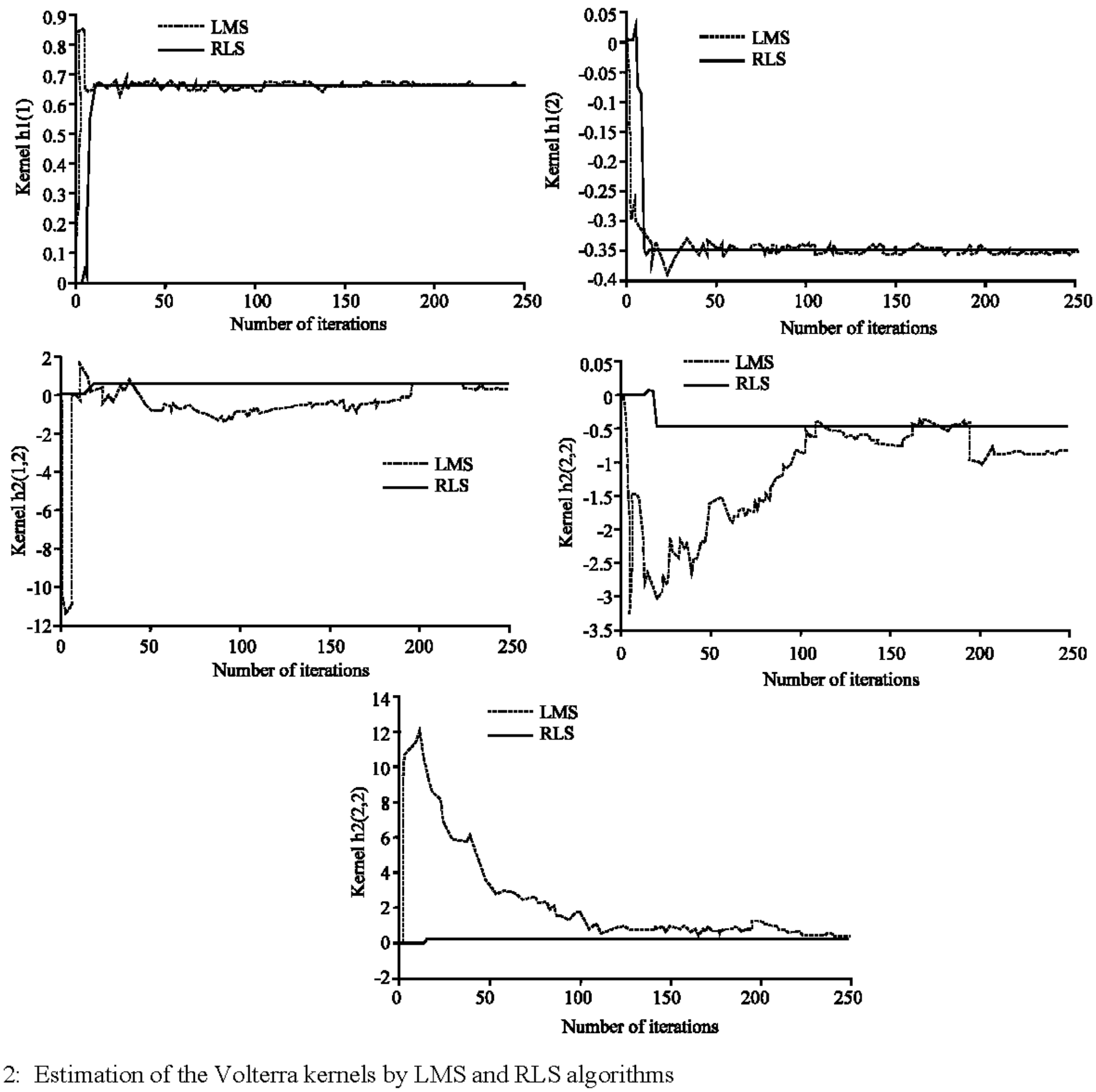

Fig. 2: Estimation of the Volterra kernels by LMS and RLS algorithms

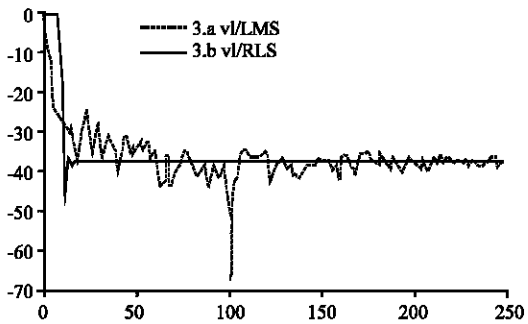

Fig. 3: Quadratic error on the estimation of linear coefficients by LMS and RLS algorithms

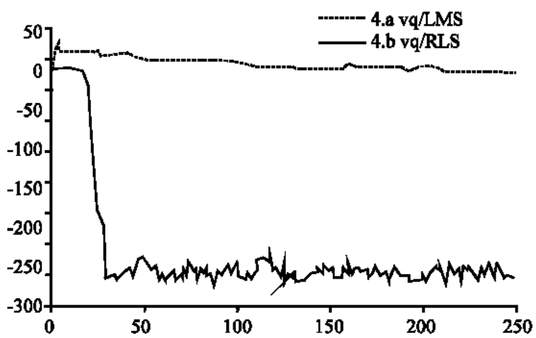

Fig. 4: Quadratic error on the estimation of quadratic coefficients by LMS and RLS algorithms

$$\| V_l[n] \| = 10\log \frac{\sum_{m_1=1}^{2}\left(\hat{h}_1[m_1,n]-h_1[n]\right)^2}{\sum_{m_1=1}^{2}\left(h_1[n]\right)^2} \qquad (10)$$

Figure 4 shows the curves of the quadratic error of quadratic coefficients by both algorithms. This error is given by Mathews and Lee[1,5]:

$$\left\| V_q[n] \right\| = 10\log \frac{\sum_{m_1=1}^{2}\sum_{m_2=m_1}^{2}\left(\hat{h}_2[m_1,m_2,n]-h_2[n]\right)^2}{\sum_{m_1=1}^{2}\sum_{m_2=m_1}^{2}\left(h_2[n]\right)^2} \qquad (11)$$

The curves pattern of Fig. 2 shows that both LMS and RLS solutions converge to the optimal solution. A divergence of LMS algorithm with important disturbances is observed on the first iterations, but the algorithm converge well in the rest of the iterations. The disturbances are negligeable for linear coefficients and important for quadratic coefficients.

The pattern of the error curves of Fig. 3a and 3b, defined by Eq. 10, shows that the LMS and RLSalgorithms perform the same for linear coefficients. This is explained by the convergence of Vl and Vrl errors with the same speed and towards the same limit-37dB.

The curves of the Fig. 4a and 4b concerning the errors on quadratic coefficients demonstrates the superiority of the RLS algorithm over the LMS algorithm regarding the quadratic coefficients. This is clear from the difference of-240dB between quadratic errors Vq and Vrq. The curves represented in Fig. 2-4 show that the RLS algorithm is more efficient than the LMS algorithm in the identification of Volterra systems.

CONCLUSION

In this study, we have shown, through a simulation study, that the RLS algorithm is more powerful than the LMS algorithm in the identification of quadratic Volterra systems. Moreover, it is shown that the adaptive noise has a direct effect on the algorithm convergence towards the optimal solution.

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